

Black Hole Solutions of Kaluza-Klein Supergravity Theories and String Theory

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ABSTRACT

We find $U(1)_E \times U(1)_M$ non-extremal black hole solutions of 6-dimensional Kaluza-Klein supergravity theories. Extremal solutions were found by Cvetič and Youm[1]. Multi black hole configurations are also presented. After electromagnetic duality transformation is performed, these multi black hole solutions are mapped into the exact solutions found by Horowitz and Tseytlin[2] in 5-dimensional string theory compactified into 4-dimensions. The massless fields of this theory can be embedded into the heterotic string theory compactified on a 6-torus. Rotating black hole solutions of this string theory can be read off from those of the heterotic string theory found by Sen[3].

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1 Introduction

There has been considerable interest in the connection between black holes and supersymmetry. The familiar example is the Reissner-Nordström black hole. This can be embedded in N=2 supergravity and the charge of the black hole is identified with the central charge of the extended supersymmetry algebra[4]. Furthermore, using methods similar to those used to prove the positivity of the ADM mass of a gravitational system[5], it was shown that the mass of the black hole is bounded from below by its charge.² This is exactly the same bound that must be satisfied for black holes to be free of naked singularities. The extremal solutions saturate the mass bound and admit a Killing spinor which is constant with respect to the supercovariant derivative. This condition gives first order differential equations to be satisfied by the extremal solutions, i.e., Killing spinor equations.

Similar phenomena have been found for other cases as well. One example is the charged black hole arising from string theory[6]. One particular feature of string theory is the nonpolynomial coupling of a scalar field to gauge fields. This is also a common characteristic of Kaluza-Klein theories[7]. Those theories have terms like $e^{2\alpha\phi} F_{\mu\nu} F^{\mu\nu}$ in the action where $F_{\mu\nu}$ is the field strength of a gauge field. Charged black hole solutions arising from such theories have attracted much attention because they have drastically different causal structures and thermodynamic properties than the Reissner-Nordström black hole. The $\alpha = 1$ case is the string theory and the supersymmetric properties of the black hole solutions in this case were studied in detail in ref.[8]. Also for $\alpha = \sqrt{3}$ the supersymmetric embedding is known and it is 5-dimensional Kaluza-Klein supergravity[9]. But it was conjectured that for an arbitrary value of α the corresponding black hole solution admits an embedding in some supergravity theory[10].

One of the motivations of the paper by Cvetič and Youm[1], is to find such embeddings

²There are some assumptions in proving this result. One of them is that the charge to mass ratio is less than or equal to 1 for any small volume of matter. Even with this assumption it is nontrivial that black hole mass is bounded below by its charge, since black holes can be formed through a complicated gravitational collapse.

for different values of α using the dimensional reduction of higher dimensional supergravity theories. They started with the minimal supersymmetric extension of pure gravity in $(4 + n)$ dimensions. Keeping only the pure gravity part and performing the dimensional reduction, they obtained the 4-dimensional theory with two Abelian gauge fields. It turns out that the resulting bosonic action for each n can be reduced to the action obtained from the 6-d Kaluza-Klein supergravity. They found the extremal black hole configurations using the Killing spinor equations.

One purpose of this paper is to find the black hole solutions of the 6-d Kaluza-Klein supergravity by directly solving the equations of motion, thereby obtaining non-extremal black hole solutions as well. This is presented in section 2. Global structures and thermal properties of the black hole solutions are explained. It turns out that the black hole solutions are intimately related with black hole solutions in string theory. We devote section 3 and section 4 to discussing those connections. In section 3 we present multi black hole solutions of the 6-d Kaluza-Klein supergravity theories. After electro-magnetic duality transformation is made, these solutions are mapped into the exact solutions of 5-d string theory compactified into 4-dimensions. This theory is considered by Horowitz and Tseytlin[2]. We briefly discuss 5-d geometry of the exact solutions. In section 4 we show that after a field redefinition, the massless fields of 5-d string theory can be embedded into the heterotic string theory. The general electrically charged, rotating black hole solutions are studied by Sen[3]. Thus we can read off the black hole solutions of the 5-d string theory from those of the heterotic string theory. In this way, we obtain the rotating black hole solutions of the 5-d string theory compactified into 4-dimensions. Conclusions and speculations are presented in section 5.

2 Black hole solutions of the 6-D Kaluza-Klein theory

The bosonic action in $4+n$ -dimension is of the form ³

$$S_{4+n} = \frac{1}{16\pi G_{4+n}} \int \sqrt{-g^{(4+n)}} d^{4+n}x R^{(4+n)}. \quad (1)$$

G_{4+n} is the gravitational constant in $4+n$ -dimension. The higher dimensional metric $g_{AB}^{(4+n)}$ is taken as

$$g_{AB}^{(4+n)} = \begin{pmatrix} e^{-\frac{1}{\alpha}\psi} g_{\lambda\pi} + e^{\frac{2\psi}{n\alpha}} \rho_{\tilde{\lambda}\tilde{\pi}} A_{\tilde{\lambda}}^{\tilde{\lambda}} A_{\tilde{\pi}}^{\tilde{\pi}} & e^{\frac{2\psi}{n\alpha}} \rho_{\tilde{\lambda}\tilde{\pi}} A_{\tilde{\lambda}}^{\tilde{\lambda}} \\ e^{\frac{2\psi}{n\alpha}} \rho_{\tilde{\lambda}\tilde{\pi}} A_{\tilde{\pi}}^{\tilde{\pi}} & e^{\frac{2\psi}{n\alpha}} \rho_{\tilde{\lambda}\tilde{\pi}} \end{pmatrix}. \quad (2)$$

Greek lower-case letters denote the space-time indices in 4-dimension while lower-case letters with tilde are used for the internal coordinates. $\rho_{\tilde{\lambda}\tilde{\pi}}$ satisfies $\det \rho_{\tilde{\lambda}\tilde{\pi}} = 1$, i.e., $\rho_{\tilde{\lambda}\tilde{\pi}}$ is the unimodular part of the internal metric. All fields have no dependence on the internal coordinates. It is shown in ref. [1] that if all the gauge fields are Abelian, the supersymmetric configuration is possible only if the internal group $U(1)^n$ is broken down to $U(1)_E \times U(1)_M$. Thus electric charge and magnetic charge should be associated to different $U(1)$ sectors. The internal metric $\rho_{\tilde{\alpha}\tilde{\beta}}$ is assumed to be

$$\rho_{\tilde{\alpha}\tilde{\beta}} = \text{diag}(\rho_1, \dots, \rho_{n-1}, \prod_{k=1}^{n-1} \rho_k). \quad (3)$$

Then the resulting action in 4-dimensions after the trivial integration over the internal coordinates is

$$S_4 = \frac{1}{16\pi G} \int \sqrt{-g} d^4x \left(R - \frac{1}{4} e^{\alpha\psi} \rho_{n-1} (F_{\mu\nu}^{n-1})^2 - \frac{1}{4} e^{\alpha\psi} \rho_n (F_{\mu\nu}^n)^2 \right. \\ \left. - \frac{1}{2} \partial_\mu \psi \partial^\mu \psi - \frac{1}{4} \sum_{i=1}^n \partial_\mu \log \rho_i \partial^\mu \log \rho_i \right), \quad (4)$$

where $\alpha = \sqrt{\frac{n+2}{n}}$. The gravitational coupling constant G in 4-dimension is given by $G = G_{n+4} (2\pi R)^n$ for a toroidal compactification where each internal dimension has radius

³With regard to the metric sign and the definition of the curvature, we follow the convention used by Misner, Thorne and Wheeler[16]. The metric signature is $(- + \dots +)$, $R_{\beta\gamma\delta}^\alpha = \partial_\gamma \Gamma_{\beta\gamma\delta}^\alpha + \dots$, $R_{\mu\nu} = R_{\mu\alpha\nu}^\alpha$.

R , $F_{\mu\nu}^{n-1}$ and $F_{\mu\nu}^n$ denote the unbroken Abelian gauge group. This action can be reduced to that of 6-d Kaluza-Klein theory with the field redefinition,

$$\begin{aligned}\phi &\equiv \frac{1}{\sqrt{2}\alpha} \psi, \quad \chi_i \equiv \frac{1}{2\sqrt{2}}(\log \rho_i + \frac{2}{n\alpha} \psi), \quad i = 1, \dots, n-2 \\ \chi_{n-1} &\equiv \frac{1}{2\sqrt{2}}(\log \rho_{n-1} + \frac{2-n}{n\alpha} \psi), \quad \chi_n \equiv \frac{1}{2\sqrt{2}}(\log \rho_n + \frac{2-n}{n\alpha} \psi).\end{aligned}\quad (5)$$

Then the corresponding action is

$$\begin{aligned}S_4 &= \frac{1}{16\pi G} \int \sqrt{-g} d^4x \left(R - \frac{1}{4} e^{2\sqrt{2}(\phi+\chi_{n-1})} (F_{n-1})^2 - \frac{1}{4} e^{2\sqrt{2}(\phi+\chi_n)} (F_n)^2 \right. \\ &\quad \left. - 2 \sum_{i=1}^n (\nabla \chi_i)^2 - 2 \partial_\mu \phi (\partial^\mu \chi_{n-1} + \partial^\mu \chi_n) - 2(\nabla \phi)^2 \right).\end{aligned}\quad (6)$$

We see that the field equations derived from this action will admit a solution with χ_i set to constant for $i = 1, \dots, n-2$. This implies that $\chi_{n-1} + \chi_n$ is also constant since $\sum_{i=1}^n \chi_i = 0$. Absorbing such constants into field redefinition of the gauge fields and defining $\chi \equiv \chi_{n-1}$, $K_{\mu\nu} \equiv F_{\mu\nu}^{n-1}$, $F_{\mu\nu} \equiv F_{\mu\nu}^n$, we finally obtain the following action ⁴

$$S = \frac{1}{16\pi} \int \sqrt{-g} d^4x \left(R - 2(\nabla \phi)^2 - 4(\nabla \chi)^2 - e^{2\sqrt{2}(\phi+\chi)} K^2 - e^{2\sqrt{2}(\phi-\chi)} F^2 \right). \quad (7)$$

If the metric in 4-dimensions takes the form

$$ds^2 = -A^2(r) dt^2 + \frac{dr^2}{A^2(r)} + R^2(r) d\Omega^2, \quad (8)$$

the equations for the gauge fields are solved by

$$K_{\theta\phi} = Q_m \sin \theta, \quad F^{rt} = \frac{Q_e}{R^2 e^{2\sqrt{2}(\phi-\chi)}}. \quad (9)$$

Then the equations of the two scalar fields are

$$\frac{1}{R^2} \frac{d}{dr} \left(R^2 A^2 \frac{d\phi}{dr} \right) = \sqrt{2} e^{2\sqrt{2}(\phi+\chi)} \frac{Q_m^2}{R^4} - \sqrt{2} e^{-2\sqrt{2}(\phi-\chi)} \frac{Q_e^2}{R^4}, \quad (10)$$

$$\frac{1}{R^2} \frac{d}{dr} \left(R^2 A^2 \frac{d\chi}{dr} \right) = \frac{\sqrt{2}}{2} e^{2\sqrt{2}(\phi+\chi)} \frac{Q_m^2}{R^4} + \frac{\sqrt{2}}{2} e^{-2\sqrt{2}(\phi-\chi)} \frac{Q_e^2}{R^4}. \quad (11)$$

⁴ From now on we set $G = c = 1$.

The gravitational field equations are

$$\begin{aligned} G_t^t &= \frac{A^2(R')^2 - 1}{R^2} + \frac{2A^2R'' + 2AA'R'}{R} \\ &= -A^2\left(\frac{d\phi}{dr}\right)^2 - 2A^2\left(\frac{d\chi}{dr}\right)^2 - e^{2\sqrt{2}(\phi+\chi)}\frac{Q_m^2}{R^4} - e^{-2\sqrt{2}(\phi-\chi)}\frac{Q_e^2}{R^4}, \end{aligned} \quad (12)$$

$$\begin{aligned} G_r^r &= \frac{A^2(R')^2 - 1}{R^2} + \frac{2AA'R'}{R} \\ &= A^2\left(\frac{d\phi}{dr}\right)^2 + 2A^2\left(\frac{d\chi}{dr}\right)^2 - e^{2\sqrt{2}(\phi+\chi)}\frac{Q_m^2}{R^4} - e^{-2\sqrt{2}(\phi-\chi)}\frac{Q_e^2}{R^4}, \end{aligned} \quad (13)$$

$$\begin{aligned} G_\theta^\theta &= \frac{1}{2}(A^2)'' + \frac{2AA'R'}{R} + \frac{A^2R''}{R} \\ &= -A^2\left(\frac{d\phi}{dr}\right)^2 - 2A^2\left(\frac{d\chi}{dr}\right)^2 + e^{2\sqrt{2}(\phi+\chi)}\frac{Q_m^2}{R^4} + e^{-2\sqrt{2}(\phi-\chi)}\frac{Q_e^2}{R^4}, \end{aligned} \quad (14)$$

where ' means the differentiation by r. From these equations, we obtain

$$R^2(G_r^r + G_\theta^\theta) = \frac{1}{2}(A^2R^2)'' - 1 = 0, \quad (15)$$

$$\begin{aligned} R^2(2G_\theta^\theta + G_r^r - G_t^t) \\ = ((A^2)'R^2)' = 2e^{2\sqrt{2}(\phi+\chi)}\frac{Q_m^2}{R^4} + 2e^{-2\sqrt{2}(\phi-\chi)}\frac{Q_e^2}{R^4}. \end{aligned} \quad (16)$$

Since we are searching for solutions with a regular horizon, we can set $A^2R^2 = (r-r_+)(r-r_-)$ with $r_+ \geq r_-$ where $r = r_+$ is the location of the horizon. If we define

$$\rho \equiv \frac{1}{r_+ - r_-} \log\left(\frac{r - r_+}{r - r_-}\right), \quad (17)$$

then we can write

$$A^2R^2\frac{d}{dr} = \frac{d}{d\rho}. \quad (18)$$

Throughout this section ρ is used as a short notation for the more complicated expression of r. Comparing the eq. (11) and eq. (16) we have

$$\frac{d^2}{d\rho^2}2\sqrt{2}\chi = \frac{d^2}{d\rho^2}\log A^2, \quad (19)$$

$$2\sqrt{2}\chi = \log A^2 + \beta\rho + 2\sqrt{2}\chi_\infty. \quad (20)$$

Here β is a constant and χ_∞ is the asymptotic value of χ . We require that χ be regular and finite at the horizon. Thus $\beta = -(r_+ - r_-)$ and

$$e^{2\sqrt{2}\chi} = A^2 \frac{r - r_-}{r - r_+} e^{2\sqrt{2}\chi_\infty}, \quad (21)$$

$$R^2 e^{2\sqrt{2}\chi} = (r - r_-)^2 e^{2\sqrt{2}\chi_\infty}, \quad (22)$$

$$\frac{R''}{R} = -\frac{2\sqrt{2}\chi'}{r - r_-} - \sqrt{2}\chi'' + 2(\chi')^2. \quad (23)$$

Subtracting (13) from (12), we obtain

$$\frac{R''}{R} = -\left(\frac{d\phi}{dr}\right)^2 - 2\left(\frac{d\chi}{dr}\right)^2. \quad (24)$$

Now the equations of the scalar fields are written as

$$\frac{d^2}{d\rho^2}(2\sqrt{2}\phi) = 4Q_m^2 A^2 e^{2\sqrt{2}(\phi+\chi)} - 4Q_e^2 A^2 e^{-2\sqrt{2}(\phi-\chi)}, \quad (25)$$

$$\frac{d^2}{d\rho^2}(4\sqrt{2}\chi) = 4Q_m^2 A^2 e^{2\sqrt{2}(\phi+\chi)} + 4Q_e^2 A^2 e^{-2\sqrt{2}(\phi-\chi)}. \quad (26)$$

Eq. (24), (25), (26) are the main equations to be considered.

If $Q_e = 0$, we see that $\phi - \phi_\infty = 2\chi - 2\chi_\infty$. Using this relation and eq. (23), eq. (24) is easily integrated to give

$$\begin{aligned} e^{2\sqrt{2}\chi} &= e^{2\sqrt{2}\chi_\infty} \sqrt{\frac{r - r_-}{r + B}}, \\ e^{2\sqrt{2}\phi} &= e^{2\sqrt{2}\phi_\infty} \frac{r - r_-}{r + B}. \end{aligned} \quad (27)$$

From eq. (21) and eq. (22) we find

$$\begin{aligned} A^2 &= \frac{r - r_+}{\sqrt{(r + B)(r - r_-)}}, \\ R^2 &= (r - r_-) \sqrt{(r + B)(r - r_-)}. \end{aligned} \quad (28)$$

From eq. (25) we can see that $(B + r_+)(B + r_-) = 4Q_m^2 e^{2\sqrt{2}(\phi_\infty + \chi_\infty)}$. We still have freedom in r-coordinate choice $r \rightarrow r + \Gamma$, using this we can set $B=0$. Since ϕ is proportional to

χ up to constant, we can define a new field ϕ' as $-2\sqrt{3}\phi' \equiv 2\sqrt{2}(\phi + \chi)$. The action in terms of ϕ' instead of ϕ and χ for $Q_e = 0$ case is

$$S = \frac{1}{16\pi} \int \sqrt{-g} d^4x (R - 2(\nabla\phi')^2 - e^{-2\sqrt{3}\phi'} K^2). \quad (29)$$

This is the action with $\alpha = \sqrt{3}$ for the Maxwell-dilaton coupling considered in [6]. Furthermore one can see that the known solution of (29) coincides with the above. Similar analysis can be performed for a purely electric case. As stated before, $\alpha = \sqrt{3}$ case corresponds to the 5-dimensional Kaluza-Klein theory, and we conclude that the solutions of a purely electric or purely magnetic case are those of 5-dimensional Kaluza-Klein supergravity theories. Also from the above analysis we know that the solution of

$$\frac{d^2}{d\rho^2}\psi = P^2 \frac{r - r_+}{r - r_-} e^{2\psi} \quad (30)$$

for constant P is given by

$$e^\psi = \frac{r - r_-}{r + B}, \quad (B + r_+)(B + r_-) = P^2. \quad (31)$$

If we consider the general case where both electric charge and magnetic charge are nonzero, eq. (25) and eq. (26) can be written as

$$\frac{d^2}{d\rho^2} \left(\frac{2\sqrt{2}\phi + 4\sqrt{2}\chi}{2} \right) = 4Q_m^2 e^{-2\sqrt{2}\chi_\infty} \frac{r - r_+}{r - r_-} e^{2\sqrt{2}\phi + 4\sqrt{2}\chi}, \quad (32)$$

$$\frac{d^2}{d\rho^2} \left(\frac{-2\sqrt{2}\phi + 4\sqrt{2}\chi}{2} \right) = 4Q_e^2 e^{-2\sqrt{2}\chi_\infty} \frac{r - r_+}{r - r_-} e^{-2\sqrt{2}\phi + 4\sqrt{2}\chi}. \quad (33)$$

These equations have the same form as (30) and this suggests that solutions have the form of (31). Hence

$$\begin{aligned} \exp \frac{2\sqrt{2}(\phi - \phi_\infty) + 4\sqrt{2}(\chi - \chi_\infty)}{2} &= \frac{r - r_-}{r + B}, \\ \exp \frac{-2\sqrt{2}(\phi - \phi_\infty) + 4\sqrt{2}(\chi - \chi_\infty)}{2} &= \frac{r - r_-}{r + C}. \end{aligned}$$

We can choose $C = -B = r_0$ by a suitable shift of $r \rightarrow r + \Gamma$. It is easily checked that this ansatz works. Thus the black hole solutions are given by

$$\begin{aligned}
A^2 &= \frac{r - r_+}{\sqrt{r^2 - r_0^2}}, \\
R^2 &= (r - r_-)\sqrt{r^2 - r_0^2}, \\
e^{2\sqrt{2}\phi} &= e^{2\sqrt{2}\phi_\infty} \frac{r + r_0}{r - r_0}, \\
e^{2\sqrt{2}\chi} &= e^{2\sqrt{2}\chi_\infty} \frac{r - r_-}{\sqrt{r^2 - r_0^2}}, \\
F^{rt} &= \frac{Q_e}{(r + r_0)^2} e^{-2\sqrt{2}(\phi_\infty - \chi_\infty)}, \\
K_{\theta\phi} &= Q_m \sin \theta.
\end{aligned} \tag{34}$$

And the constraints on r_+, r_-, r_0 are

$$\begin{aligned}
(r_+ + r_0)(r_- + r_0) &= 4Q^2, \\
(r_+ - r_0)(r_- - r_0) &= 4P^2.
\end{aligned} \tag{35}$$

Here P and Q are defined as

$$\begin{aligned}
Q^2 &\equiv Q_e^2 e^{-2\sqrt{2}\phi_\infty + 2\sqrt{2}\chi_\infty}, \\
P^2 &\equiv Q_m^2 e^{2\sqrt{2}\phi_\infty + 2\sqrt{2}\chi_\infty}.
\end{aligned} \tag{36}$$

From solution (34), we see that $r_+ = 2M$ where M is the mass of a black hole, $r_0 = \sqrt{2}\Sigma$ where Σ is the charge of the scalar ϕ , and $r_- = -2\Delta$ where Δ is the charge of χ . The charge Σ is defined by

$$\phi = \phi_\infty + \frac{\Sigma}{r} + O\left(\frac{1}{r^2}\right), \quad r \rightarrow \infty. \tag{37}$$

On the other hand, Δ is defined by

$$\sqrt{2}\chi = \sqrt{2}\chi_\infty + \frac{\Delta}{r} + O\left(\frac{1}{r^2}\right), \quad r \rightarrow \infty, \tag{38}$$

since we adopt the different normalization for ϕ and χ in action (7). Clearly Σ and Δ are not independent parameters and depend on M, P and Q . Their dependence is given by cubic equations but those equations are not particularly illuminating. Since P and Q depend on Q_e, Q_m and ϕ_∞, χ_∞ in turn, the black hole solutions are characterized by five parameters i.e., mass, electric charge, magnetic charge, and asymptotic values of scalar fields. From eq. (35), one can easily find that $M \geq \frac{|Q|+|P|}{2}$ and $|\Sigma| \leq \frac{||Q|-|P||}{\sqrt{2}}$. The sign of Σ is the same as that of $|Q| - |P|$. Also $r_+ \geq r_- \geq |r_0|$. Calculation of the curvature tensors indicates that $r = r_+$ is indeed a regular horizon and $r = r_-$ is a curvature singularity. Comparing with the Reissner-Nordström black hole, the would-be inner horizon turns into the singularity. The extremal solutions where r_+ coincides with r_- agree with the solutions found by Cvetič and Youm[1]. Note that $M = \frac{|Q|+|P|}{2}$, $\Sigma = \frac{|Q|-|P|}{\sqrt{2}}$ and $\Delta = -M$ so that $M^2 + \Sigma^2 + \Delta^2 = Q^2 + P^2$ in extremal case. This is the force balance condition, as we will see later.

The causal structure of the non-extremal case is that of the Schwarzschild black hole. For extremal black holes, the situation is two-fold. If both electric and magnetic charge are non-zero, the corresponding black hole has a null singularity. This can be seen from the fact that the radial null geodesics satisfy $\pm dt \propto dr/(r-r_+)$, which implies that as $r \rightarrow r_+$, the geodesics reach arbitrarily large values of $|t|$. This shows that an outgoing null geodesic must cross every ingoing null geodesic. However, if either of the charges is zero, the singularity becomes timelike naked. In this case $r_+ = r_- = \pm r_0$ and $\pm dt \propto dr/\sqrt{r-r_+}$ for radial null rays near $r = r_+$. The Penrose diagrams are given for each case in Figure 1.

The temperature of the black holes can be found from the periodicity of its Euclidean continuation[11], or alternatively from its surface gravity. It is $T = \frac{1}{4\pi\sqrt{r_+^2 - r_0^2}}$. In the extremal case, $r_+ = |Q| + |P|$ and $r_- = |Q| - |P|$. Hence T approaches $\frac{1}{8\pi\sqrt{|P||Q|}}$ in the extremal limit. The entropy of the black holes can be evaluated in two ways. One may integrate the first law of thermodynamics. Or one can calculate the thermodynamic

functions directly, using the saddle point approximation for the action of the black holes in the Euclidean continuation[11]. Either way gives $S = \frac{1}{4}A = \pi(r_+ - r_-)\sqrt{r_+^2 - r_0^2}$. In the extremal limit, the black holes approach zero entropy and nonzero temperature configurations.⁵

So far we have presented the black hole solutions of 6-d Kaluza-Klein supergravity. For other dimensions we can read off the result from eq. (5). However the same metric remains a solution for all dimensions.

3 Multi black hole solutions and their string interpretation

Now we will look for a static solution representing a collection of extremal black holes, with the following ansatz for the metric in isotropic coordinates

$$ds^2 = -e^{2U} dt^2 + e^{-2U} (dx_i)^2. \quad (39)$$

The nonzero components of the Ricci tensor in the coordinate basis are

$$\begin{aligned} R_{00} &= e^{4U} \partial_i \partial_i U \\ R_{ij} &= -2\partial_i U \partial_j U + \delta_{ij} \partial_k \partial_k U. \end{aligned} \quad (40)$$

If we choose the gauge fields as

$$F_{i0} = \partial_i \Psi, \quad K_{jk} = \varepsilon_{ijk} e^{-2\sqrt{2}(\phi+\chi)-2U} \partial_k \lambda, \quad (41)$$

the equations of motion and Bianchi identities give

$$\begin{aligned} \partial_i (e^{2\sqrt{2}(\phi-\chi)-2U} \partial_i \Psi) &= 0, \\ \partial_i (e^{-2\sqrt{2}(\phi+\chi)-2U} \partial_i \lambda) &= 0. \end{aligned} \quad (42)$$

⁵There are claims that the extremal black holes always have zero entropy and can be in equilibrium with thermal radiation at any temperature[12]. This indicates that we have to distinguish between an extremal black hole and limiting configuration from a non-extremal black hole, if the claim is right.

And the scalar field equations are

$$\begin{aligned}\partial_i \partial_i \phi &= \sqrt{2} e^{-2\sqrt{2}(\phi+\chi)-2U} (\partial_i \lambda)^2 - \sqrt{2} e^{2\sqrt{2}(\phi-\chi)-2U} (\partial_i \Psi)^2, \\ \partial_i \partial_i \chi &= \frac{\sqrt{2}}{2} e^{-2\sqrt{2}(\phi+\chi)-2U} (\partial_i \lambda)^2 + \frac{\sqrt{2}}{2} e^{2\sqrt{2}(\phi-\chi)-2U} (\partial_i \Psi)^2.\end{aligned}\quad (43)$$

The gravitational field equations give

$$\partial_i \partial_i U = e^{-2\sqrt{2}(\phi+\chi)-2U} (\partial_i \lambda)^2 + e^{2\sqrt{2}(\phi-\chi)-2U} (\partial_i \Psi)^2, \quad (44)$$

$$\partial_i U \partial_j U = -\partial_i \phi \partial_j \phi - 2\partial_i \chi \partial_j \chi + e^{-2\sqrt{2}(\phi+\chi)-2U} \partial_i \lambda \partial_j \lambda + e^{2\sqrt{2}(\phi-\chi)-2U} \partial_i \Psi \partial_j \Psi. \quad (45)$$

The relevant solutions of these equations are

$$\begin{aligned}\sqrt{2}\chi &= U + \sqrt{2}\chi_\infty, \\ e^{-\sqrt{2}(\phi+2\chi)} &= H_1, \quad e^{-\sqrt{2}(-\phi+2\chi)} = H_2, \\ 2e^{\sqrt{2}\chi_\infty} \lambda &= \pm \frac{1}{H_1}, \quad 2e^{\sqrt{2}\chi_\infty} \Psi = \pm \frac{1}{H_2}, \\ \partial_i \partial_i H_1 &= 0, \quad \partial_i \partial_i H_2 = 0.\end{aligned}\quad (46)$$

One particular solution representing the multiblack hole configuration is given by

$$\begin{aligned}H_1 &= e^{-\sqrt{2}(\phi_\infty+2\chi_\infty)} \left(1 + \sum_{i=1}^n \frac{2|P_i|}{|x-x_i|}\right), \\ H_2 &= e^{\sqrt{2}(\phi_\infty-2\chi_\infty)} \left(1 + \sum_{i=1}^n \frac{2|Q_i|}{|x-x_i|}\right).\end{aligned}\quad (47)$$

One can easily see that this is the collection of n extremal black holes. The relation between the parameters of each black hole is

$$\begin{aligned}M_i &= -\Delta_i = \frac{|Q_i| + |P_i|}{2}, \\ \Sigma_i &= \frac{|Q_i| - |P_i|}{\sqrt{2}}.\end{aligned}\quad (48)$$

This condition implies the force balance. To see this explicitly, let us consider gravitational, electromagnetic, and scalar forces. The force between two distant objects of masses and charges $(M_1, Q_1, P_1, \Sigma_1, \Delta_1)$ and $(M_2, Q_2, P_2, \Sigma_2, \Delta_2)$ is given by

$$F_{12} = -\frac{M_1 M_2}{r_{12}^2} + \frac{Q_1 Q_2}{r_{12}^2} + \frac{P_1 P_2}{r_{12}^2} - \frac{\Sigma_1 \Sigma_2}{r_{12}^2} - \frac{\Delta_1 \Delta_2}{r_{12}^2}. \quad (49)$$

The scalar forces are attractive for charges of the same type and repulsive for charges of opposite sign. Using eq. (48), we see that F_{12} vanishes. This force balance allows the black holes to be located at any place, in equilibrium with the other black holes.

Interestingly enough, the above solutions are related to the exact string solutions found by Horowitz and Tseytlin[2] by the electromagnetic dual transformation. From the action (7), if we perform the duality transformation

$$K_{\mu\nu} = \frac{1}{2}\varepsilon_{\mu\nu\lambda\sigma}C^{\lambda\sigma}e^{-2\sqrt{2}(\phi+\chi)}, \quad (50)$$

the resulting action is

$$S_1 = \frac{1}{16\pi} \int \sqrt{-g} d^4x (R - 2(\nabla\phi)^2 - 4(\nabla\chi)^2 - e^{-2\sqrt{2}(\phi+\chi)}C^2 - e^{2\sqrt{2}(\phi-\chi)}F^2), \quad (51)$$

where $\varepsilon_{\mu\nu\lambda\sigma}$ is an antisymmetric tensor with $\varepsilon_{1234} = 1$. Now if we define

$$\begin{aligned} F_{\mu\nu}^s &\equiv 2F_{\mu\nu}, & B_{\mu\nu}^s &\equiv 2C_{\mu\nu}, \\ \varphi &\equiv 2\sqrt{2}\chi, & \sigma &\equiv \sqrt{2}\phi, \end{aligned} \quad (52)$$

the action is written as

$$S_2 = \frac{1}{16\pi} \int \sqrt{-g} d^4x (R - (\nabla\sigma)^2 - \frac{1}{2}(\nabla\varphi)^2 - \frac{1}{4}e^{-2\sigma-\varphi}(B_{\mu\nu}^s)^2 - \frac{1}{4}e^{2\sigma-\varphi}(F_{\mu\nu}^s)^2). \quad (53)$$

This is the dimensional reduction of the 5-d bosonic string action. To see this, we start with the leading-order term in 5-d bosonic string action

$$S_5 = \kappa^0 \int d^5x \sqrt{-g_{(5)}} e^{-2\phi_s} (R + 4(\nabla\phi_s)^2 - \frac{1}{12}(H_{MNK})^2 + o(\alpha')), \quad (54)$$

where $H_{MNK} = 3\partial_{[M}B'_{NK]} = \partial_MB'_{NK} + \partial_KB'_{MN} + \partial_NB'_{KM}$. Here $\partial_{[M}B'_{NK]}$ is an antisymmetric third rank tensor of strength 1.⁶ Assuming that all the fields are independent of x^5 , we obtain the 4-d reduced action

$$\begin{aligned} \tilde{S}_4 &= \hat{\kappa}_0 \int d^4x \sqrt{-g} e^{-2\phi_s+\sigma} (\hat{R} + 4(\partial_\mu\phi_s)^2 - 4\partial_\mu\phi_s\partial^\mu\sigma \\ &\quad - \frac{1}{12}(\hat{H}_{\mu\nu\lambda})^2 - \frac{1}{4}e^{2\sigma}(F_{\mu\nu}^s)^2 - \frac{1}{4}e^{-2\sigma}(B_{\mu\nu}^s)^2 + o(\alpha')) \end{aligned} \quad (55)$$

⁶From now on we denote an antisymmetric n -th rank tensor of strength 1 as $A_{[\mu_1\mu_2\cdots\mu_n]}$.

where

$$\begin{aligned}
g_{55} &\equiv e^{2\sigma}, & F_{\mu\nu}^s &= \partial_\mu A_\nu^s - \partial_\nu A_\mu^s, \\
B_{\mu\nu}^s &= \partial_\mu B_\nu^s - \partial_\nu B_\mu^s, & A_\mu^s &\equiv g_{\mu 5} e^{-2\sigma}, \\
B_\mu^s &\equiv B'_{\mu 5}, & \hat{H}_{\lambda\mu\nu} &= 3\partial_{[\lambda} B'_{\mu\nu]} - 3A_{[\lambda}^s B_{\mu\nu]}^s.
\end{aligned} \tag{56}$$

5-d metric is given in terms of 4-d metric as

$$ds^2 = e^{2\sigma}(dx^5 + A_t dt)^2 + g_{\alpha\beta} dx^{\alpha\beta}. \tag{57}$$

Setting $\varphi = 2\phi - \sigma$ and using the Einstein metric $g_{\alpha\beta}^E = e^{-\varphi} g_{\alpha\beta}$, we obtain

$$\begin{aligned}
S'_4 &= \hat{\kappa}_0 \int \sqrt{-g_E} d^4x (R_E - (\nabla\sigma)^2 - \frac{1}{2}(\nabla\varphi)^2 - \frac{1}{12}e^{-2\varphi}(\hat{H}_{\mu\nu\lambda})^2 \\
&\quad - \frac{1}{4}e^{-2\sigma-\varphi}(B_{\mu\nu}^s)^2 - \frac{1}{4}e^{2\sigma-\varphi}(F_{\mu\nu}^s)^2 + o(\alpha')).
\end{aligned} \tag{58}$$

This is equal to S_2 .

Using the relation (52) we can see that the black hole solutions for the 6-d Kaluza Klein supergravity theory are the solutions of S'_4 with $\hat{H}_{\mu\nu\lambda} = 0$ if expressed in the new variables. For later use we present a black hole solution. It is given by

$$ds_E^2 = -\frac{r-r_+}{\sqrt{r^2-r_0^2}}dt^2 + \frac{\sqrt{r^2-r_0^2}}{r-r_+}dr^2 + (r-r_-)\sqrt{r^2-r_0^2}d\Omega^2, \tag{59}$$

$$\begin{aligned}
\exp \varphi &= \frac{r-r_-}{\sqrt{r^2-r_0^2}}, & \exp \sigma &= \sqrt{\frac{r+r_0}{r-r_0}}, \\
A_t &= -\frac{Q}{r+r_0}, & B_t &= -\frac{P}{r-r_0},
\end{aligned}$$

with

$$\begin{aligned}
(r_+ - r_0)(r_- - r_0) &= P^2, \\
(r_+ + r_0)(r_- + r_0) &= Q^2.
\end{aligned} \tag{60}$$

Here we set $\varphi_\infty = \sigma_\infty = 0$. For an extremal black hole, we can use the expressions (46) and (47). The result is

$$ds_E^2 = -\frac{\rho}{\sqrt{(\rho + |P|)(\rho + |Q|)}} dt^2 + \sqrt{(1 + \frac{|P|}{\rho})(1 + \frac{|Q|}{\rho})} (d\rho^2 + \rho^2 d\Omega^2), \quad (61)$$

$$e^\varphi = \frac{\rho}{\sqrt{(\rho + |P|)(\rho + |Q|)}}, \quad e^\sigma = \sqrt{\frac{\rho + |Q|}{\rho + |P|}},$$

$$A'_t = \frac{\rho}{\rho + |Q|}, \quad B'_t = \frac{\rho}{\rho + |P|}.$$

In order to obtain (61) from (59), we can use $r_+ - r_0 = |P|$ and $r_+ + r_0 = |Q|$ for extremal case and define $r - r_+ \equiv \rho$ and we recover (61). But we need a constant shift of the vector potential which is a gauge transformation. Concretely,

$$A'_t - A_t = \frac{\rho}{\rho + |Q|} + \frac{Q}{\rho + r_+ + r_0} = \frac{\rho + Q}{\rho + |Q|} = 1 \quad (62)$$

if $Q \geq 0$. And similarly $B'_t = B_t + 1$ if $P \geq 0$.

The multi black hole solutions found at (46) and (47) are transformed into the solutions found by Horowitz and Tseytlin. ((4.6) in their paper[2].) Those solutions are exact solutions to all orders in α' . These bosonic solutions are also shown to be exact in the closed superstring and in the heterotic string theory as well.

For one black hole configuration in 4-dimensions, the corresponding 5-metric is

$$ds^2 = \frac{\rho + |Q|}{\rho + |P|} (dx^5 + \frac{\rho}{\rho + |Q|} dt)^2 - \frac{\rho^2 dt^2}{(\rho + |P|)(\rho + |Q|)} + d\rho^2 + \rho^2 d\Omega^2 \quad (63)$$

$$= \frac{\rho + |Q|}{\rho + |P|} (dx^5)^2 + \frac{2\rho}{\rho + |P|} dx^5 dt + d\rho^2 + \rho^2 d\Omega^2.$$

Here the internal coordinate x^5 becomes a null coordinate in this extremal limit and the 5-dimensional metric describes the gravitational plane wave. If $|P| = 0$, the metric describes the usual 5-d geometry of Kaluza-Klein electric black hole. The wavelike behavior can be understood in the following heuristic way. One can obtain this solution by boosting the Schwarzschild solution into the fifth dimension. This still satisfies the 5-dimensional

field equation. However when reinterpreted in 4-dimension, one obtains a solution with a nonzero Maxwell field and dilaton. These solutions are regular if the velocity in the fifth direction is subliminal. As it tends to that of light, they become singular. In this limit, after simultaneously rescaling the size we obtain the above solution with $|P| = 0$, which describe pointlike singularities moving with the velocity of light. It is not clear whether a similar interpretation is possible for $|P||Q| \neq 0$.

4 Rotating black hole solutions of the 5-dimensional string theory compactified into 4-dimensions

There is a close relationship between the 5-dimensional string theory compactified into 4-dimensions, and 4-dimensional heterotic string theory with toroidal compactification. If we define $\Phi = 2\phi_s - \sigma$, then \tilde{S}_4 in (55) is written as

$$\tilde{S}_4 = \int d^4x \sqrt{-g} e^{-\Phi} (R + (\partial_\mu \Phi)^2 - (\partial_\mu \sigma)^2 - \frac{1}{4} e^{2\sigma} (F_{\mu\nu}^s)^2 - \frac{1}{4} e^{-2\sigma} (B_{\mu\nu}^s)^2) \quad (64)$$

On the other hand, the massless fields in heterotic string theory compactified on a six dimensional torus consists of the metric $g_{\mu\nu}$, the antisymmetric tensor field $B_{\mu\nu}$, 28 $U(1)$ gauge fields $A_\mu^{(a)}$ ($1 \leq a \leq 28$), the scalar dilaton field Φ , and a 28×28 symmetric matrix valued scalar field M satisfying,

$$MLM^T = L, \quad M^T = M. \quad (65)$$

Here L is a 28×28 symmetric matrix with 22 eigenvalues -1 and 6 eigenvalues $+1$. For definiteness we will take L to be

$$L = \begin{pmatrix} -I_{22} & \\ & I_6 \end{pmatrix}, \quad (66)$$

where I_n denotes an $n \times n$ identity matrix. The action describing the effective field theory of these massless bosonic fields is given by [3],

$$S = C \int d^4x \sqrt{-g} e^{-\Phi} (R + (\partial_\mu \phi)^2 + \frac{1}{8} \text{Tr}(\partial_\mu M L \partial^\mu L) - F_{\mu\nu}^{(a)} (L M L)_{ab} F^{\mu\nu(b)} - \frac{1}{12} H_{\mu\nu\rho} H^{\mu\nu\rho}), \quad (67)$$

where,

$$\begin{aligned} F_{\mu\nu}^{(a)} &= \partial_\mu A_\nu^{(a)} - \partial_\nu A_\mu^{(a)}, \\ H_{\mu\nu\rho} &= 3\partial_{[\mu} B_{\nu\rho]} + 6A_{[\mu}^{(a)} F_{\nu\rho]}^{(b)} L_{ab}. \end{aligned} \quad (68)$$

If we choose the special M

$$M = \begin{pmatrix} \cosh 2\sigma & & \sinh 2\sigma \\ & -I_{21} & \\ \sinh 2\sigma & & \cosh 2\sigma \\ & & & I_5 \end{pmatrix}, \quad (69)$$

then S can be reduced to

$$\begin{aligned} S_4 &= \int d^4x \sqrt{-g} e^{-\Phi} (R + (\partial_\mu \Phi)^2 - (\partial_\mu \sigma)^2 \\ &\quad - e^{2\sigma} \left(\frac{F_{\mu\nu}^{(1)} - F_{\mu\nu}^{(23)}}{\sqrt{2}} \right)^2 - e^{-2\sigma} \left(\frac{F_{\mu\nu}^{(1)} + F_{\mu\nu}^{(23)}}{\sqrt{2}} \right)^2). \end{aligned} \quad (70)$$

Thus if we set

$$\begin{aligned} F_{\mu\nu}^s &= \sqrt{2}(F_{\mu\nu}^{(1)} - F_{\mu\nu}^{(23)}), \\ B_{\mu\nu}^s &= \sqrt{2}(F_{\mu\nu}^{(1)} + F_{\mu\nu}^{(23)}), \end{aligned} \quad (71)$$

we recover the action (64). Hence we can read off the black hole solutions of (64) from Sen's results, which construct the general electrically charged rotating black hole solutions of the heterotic string theory.

But there is a little difference in the definition of the antisymmetric tensor fields between Sen's work and Horowitz and Tseytlin's. Following the Sen's definition (68), we obtain

$$H_{\mu\nu\lambda} = 3\partial_{[\mu} B_{\nu\lambda]} - \frac{3}{2}A_{[\mu}^s B_{\nu\lambda]}^s - \frac{3}{2}B_{[\mu}^s F_{\nu\lambda]}^s, \quad (72)$$

while Horowitz and Tseytlin use

$$\hat{H}_{\mu\nu\lambda} = 3\partial_{[\mu} B'_{\nu\lambda]} - 3A_{[\mu}^s B_{\nu\lambda]}^s. \quad (73)$$

If we set $B'_{\mu\nu} = B_{\mu\nu} - A_{[\mu}^s B_{\nu]}^s$, $H_{\mu\nu\lambda}$ is equal to $\hat{H}_{\mu\nu\lambda}$. Thus two definitions differ by field redefinition and this redefinition does not change the gauge invariant field strength. One can check that equations of motion do not change under the above field redefinition. The rotating black hole solutions are given by

$$\begin{aligned}
ds_E^2 = & -\frac{\rho^2 + a^2 \cos^2 \theta - 2m\rho}{\sqrt{\Delta}} dt^2 + \frac{\sqrt{\Delta}}{\rho^2 + a^2 - 2m\rho} d\rho^2 + \sqrt{\Delta} d\theta^2 \\
& + \frac{\sin^2 \theta}{\sqrt{\Delta}} [\Delta + a^2 \sin^2 \theta (\rho^2 + a^2 \cos^2 \theta + 2m\rho \cosh \alpha \cosh \beta)] d\phi^2 \\
& - \frac{2}{\sqrt{\Delta}} m\rho a \sin^2 \theta (\cosh \alpha + \cosh \beta) dt d\phi,
\end{aligned}$$

where $ds_E^2 = e^{-\Phi} g_{\alpha\beta} dx^\alpha dx^\beta$ is the Einstein metric and

$$\Delta = [\rho^2 + a^2 \cos^2 \theta + m\rho(\cosh \alpha \cosh \beta - 1)]^2 - m^2 \rho^2 \sinh^2 \alpha \sinh^2 \beta. \quad (74)$$

The other fields are given by

$$\begin{aligned}
\exp \Phi &= \frac{\rho^2 + a^2 \cos^2 \theta}{\Delta} \\
\exp \sigma &= \frac{\rho^2 + a^2 \cos^2 \theta + m\rho(\cosh \alpha \cosh \beta - \sinh \alpha \sinh \beta - 1)}{\Delta}
\end{aligned} \quad (75)$$

$$\begin{aligned}
A_t^s &= -\Delta^{-1} m\rho [(\rho^2 + a^2 \cos^2 \theta)(\cosh \beta \sinh \alpha - \cosh \alpha \sinh \beta) \\
&\quad + m\rho(\cosh \alpha - \cosh \beta)(\sinh \alpha + \sinh \beta)]
\end{aligned} \quad (76)$$

$$\begin{aligned}
A_\phi^s &= \Delta^{-1} m\rho a \sin^2 \theta [(\rho^2 + a^2 \cos^2 \theta)(\sinh \alpha - \sinh \beta) \\
&\quad + m\rho(\cosh \alpha - \cosh \beta)(\sinh \alpha \cosh \beta + \sinh \beta \cosh \alpha)],
\end{aligned}$$

$$\begin{aligned}
B_t^s &= -\Delta^{-1} m\rho [(\rho^2 + a^2 \cos^2 \theta)(\cosh \beta \sinh \alpha + \cosh \alpha \sinh \beta) \\
&\quad + m\rho(\cosh \alpha - \cosh \beta)(\sinh \alpha - \sinh \beta)],
\end{aligned} \quad (77)$$

$$\begin{aligned}
B_\phi^s &= \Delta^{-1} m\rho a \sin^2 \theta [(\rho^2 + a^2 \cos^2 \theta)(\sinh \alpha + \sinh \beta) \\
&\quad + m\rho(\cosh \alpha - \cosh \beta)(\sinh \alpha \cosh \beta - \sinh \beta \cosh \alpha)],
\end{aligned}$$

$$B_{t\phi} = \Delta^{-1} m \rho a \sin^2 \theta (\cosh \alpha - \cosh \beta) [(\rho^2 + a^2 \cos^2 \theta) + m \rho (\cosh \alpha \cosh \beta - 1)],$$

and $B'_{\mu\nu} = B_{\mu\nu} - A_{[\mu}^s B_{\nu]}^s$. The various properties of the above solutions are studied at [3]. Non-extremal solutions with non-zero angular momentum have two horizons and their global structure are similar to that of the Kerr black hole solutions. The extremal limit with non-zero angular momentum has non-zero surface area and zero temperature. When $a = 0$, the above solution describes spherically symmetric black holes. If we set

$$\begin{aligned} r &\equiv \rho + m(\cosh \alpha \cosh \beta - 1), & r_+ &= m(1 + \cosh \alpha \cosh \beta) \\ r_- &= m(\cosh \alpha \cosh \beta - 1), & r_0 &= -m \sinh \alpha \sinh \beta \\ Q &= m(\sinh \alpha \cosh \beta - \sinh \beta \cosh \alpha), & P &= m(\sinh \alpha \cosh \beta + \sinh \beta \cosh \alpha) \end{aligned} \quad (78)$$

we obtain the previous solution (59).

5 Discussion

We started with black hole solutions of 6-d Kaluza-Kline theory and found interesting connections of those solutions with string theories. Two kinds of string theories are mainly discussed. One is 5-d bosonic string theory compactified into 4-dimensions, and the other is 4-d heterotic string theory with toroidal compactification. Actually, black hole solutions of 6-d Kaluza-Klein theory and of 5-d string theory can be read off from those of the heterotic string theory. Black hole solutions of 5-d string theory can also be embedded into the closed superstring theory. It is not surprising that such connection between Kaluza-Klein black hole solutions and string theories. Many of supergravity theories can be obtained by dimensional reduction of underlying supergravity theory of the closed superstring theory or the heterotic string theory with consistent truncation of some fields. Thus the embedding of massless fields of 5-d string theory can be regarded as the embedding of the underlying supergravity theory into string theories.

Once black hole solutions of 5-d string theory is embedded into the heterotic string theory or the closed string theory, we can generate other solutions using T-duality trans-

formations. However, it's not clear that the transformed solutions are also exact solutions of the underlying string theory since the leading order duality transformation can receive α' corrections. It remains to be seen if a similar argument of exactness can be given to the transformed solutions as Horowitz and Tseytlin did.

As this work is completed, we receive a preprint by M. Cvetič et. al. [18] which worked out non-extremal solutions of 6-d Kaluza-Klein theory independently. M. Cvetič informed the author that the non-extremal solutions were presented in another preprint[19] in December, 94. But this preprint was not submitted to the hep-th bulletin board. The author thanks to her for sending this preprint to him.

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Figure Caption

Fig.1: Penrose diagrams for the black hole solutions. Fig.1a is for the non-extremal case, Fig.1b is for extremal black holes with both charges non zero and Fig.1c corresponds to extremal black holes with only one charge.